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STUDY OF A NUCLEAR GAMMA-RAY LASER(U) CORNELL UNIV  
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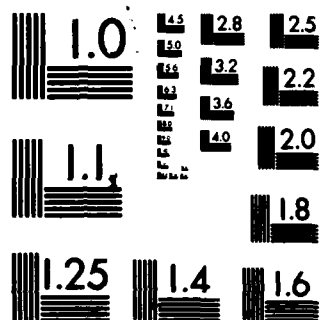
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Authors: R. L. Liboff, D. M. Heffernan and  
B. Yaakobi

Principal Investigator: Richard L. Liboff

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) <b>Two radioactive nuclear decay schemes are examined as possible <math>\gamma</math>-ray lasing devices. These involve certain relatively long-lived first excited states above the ground state and higher lying iso- meric states. The population of such excited states, neglecting stimulated decay, is found to vary as <math>(\tau_s/\tau_2)N_2(0)[1 - \exp(-t/\tau_s)]</math> where <math>\tau_s</math> is the decay time of the sth excited state of the lasing nucleus and <math>\tau_2</math> is the decay time of the parent species (continued)</b>		

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20. Abstract (continued)

whose initial concentration is  $N_\ell(0)$ . For decay to the stable nuclei  $^{161}_{66}\text{Dy}$  and  $^{191}_{77}\text{Ir}$ , the decay ratio  $\tau_s/\tau_\ell$  is found to be infinitesimally small. A brief discussion of the emitted multipole fields is included.

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## 1. Introduction

In this analysis we describe briefly a possible approach to the realization of a  $\gamma$ -ray laser ("graser")<sup>1,2</sup>. The proposed scheme has its basis in certain long-lived excited states among the heavier nuclei. Decay of such states, for the most part, gives rise to high-order multipole radiation. These excited states are populated through either  $\beta$ -decay or electron capture. The device presumes that a collection of such long-lived excited states is unstable to induced resonant emission. Decay of parent nuclei serve as the pumping mechanism in the proposed lasing scheme.

Two decay modes are examined. The first concerns the lowest lying state above the ground state, which in many cases, is long-lived compared to the higher lying states. A portion of these higher lying states serves to populate the first excited state. Analysis of population growth applied specifically to the decay of the first excited state of  ${}^{161}_{66}\text{Dy}$ , corresponding to 25.7 keV, reveals an infinitesimal growth of this excited state.

The second decay mode studied addresses higher lying isomeric states. Whereas population growth of such states is enhanced owing to the long life times of these states, it is still rendered small by the very long life times of the parent nuclei. Specific attention here is given to the excited states of  ${}^{191}_{77}\text{Ir}$ . Transition between the third and second excited states of this nucleus gives rise to X radiation of wavelength 0.028 Å. Other means of exciting isomeric states have been discussed in the literature<sup>3,4,5</sup>. These include population through fast and slow neutron bombardment and fast neutron pumping of a Mossbauer crystal.

Multipole emission occurs in addition to internal conversion<sup>6</sup>, in which an electron is ejected from the atom. Internal conversion in turn, is accompanied by atomic X-radiation. The ratio of transition probability for internal conversion to  $l$ -multipole emission from a nucleus of charge  $Z$  is given by

$$\alpha_l = \frac{16}{(2l+1)^3} \frac{l}{l+1} Z^{-1} \left( \frac{Ze^2}{\hbar c} \frac{mc^2}{\hbar \omega} \right)^{2l+1} ka$$

where  $a$  is the radius of the K or L orbit and  $k$  is the wave number of the field. This result is valid for  $ka \ll 1$ .<sup>7</sup>

## 2. Nuclear Decay Schemes

We consider two mechanisms for populating the excited states of the nucleus  $Z+1y^M$ . The first of these is through  $\beta$  decay:



Electron capture (EC) of the adjoining parent nucleus  $Z+2w^M$  also serves to populate the excited states of  $Z+1y^M$ .



A schematic<sup>8</sup> of the processes (1) and (2) is shown in Fig. 1. The sequence  $\{s_i\}$  represents excited states of  $Z+1y^M$ . In many such decay configurations the lifetime of the first excited state of  $Z+1y^M$  is in excess of higher lying excited states. Furthermore, a portion of the higher lying states decay to  $s_1$  in a time short compared to the lifetime of  $s_1$ . Let us call such states  $s'_i$ . These short-lived states are also

populated directly through decay of the adjoining parent nuclei,  $Zx^M$  and  $Z+2w^M$ .

### 3. Two Decay Schemes

In our first proposed scheme, the lasing frequency corresponds to the decay of the first excited state of  $Z+1y^M$  to the ground state. The frequency of this radiation is given by the Bohr formula,

$$\hbar\omega = E_{s_1} - E_{s_0} \quad (3)$$

The conjectured lasing configuration is comprised of a shielded mixture of  $Zx^M$  and  $Z+2w^M$ . If a population of excited states of  $Z+1y^M$  were to develop, it is hypothesized that photons from spontaneous decay of  $s_1$  to  $s_0$  would induce decay of the long-lived  $s_1$  state in a neighboring nucleus thereby effecting a lasing scheme.

The second approach emerges from like decay schemes involving extremely long-lived isomeric nuclear states. Such states are found in nuclei with neutron or proton numbers immediately below the nucleon- 'magic numbers', ( $N = 2, 8, 20, 28, 50, 82, 126$ ) thereby allowing for large nuclear angular momentum and long life times<sup>9</sup>. These observations are found to be consistent with the so-called independent-particle model of nuclei<sup>10,11</sup>. Again, these isomeric states are populated through  $\beta$  decay from the parent nucleus  $Zx^M$  or through electron capture from the parent nucleus  $Z+2w^M$ . The decay of such isomeric states is typically to adjacent lower lying states. The large angular momentum of such states gives rise to high order multipole

radiation. For close lying adjacent levels, radiation lies in the X-ray domain.

#### 4. Population Growth

Let us ascertain the population density of s-excited states,  $N_y^s(t)$ , of stable y nuclei at the time t. Labeling the number of parent nuclei: x, w, etc., as  $N_l(t)$ , we write,

$$\frac{dN_y^s}{dt} = -\nu_s N_y^s - \kappa n(\omega) N_y^s + \sum_{l=1}^L f_l^s \nu_l N_l(t) \quad (4)$$

In this expression  $n(\omega)$  denotes photon number density at the lasing frequency  $\omega$ . Inverse decay time of the sth excited state is  $\nu_s$  and  $\nu_l$  is inverse decay time of the lth parent nucleus. The fraction of decay nuclei which contribute to  $N_y^s$  is written  $f_l^s$ . The resonant photon density grows at a rate proportional to the decay rate of  $N_y^s$  and the density of excited states  $N_y^s$ .

$$\frac{dn(\omega)}{dt} = \alpha N_y^s \frac{dN_y^s}{dt} \quad (5)$$

The constant  $\kappa$  has dimensions  $\text{cm}^3/\text{sec}^2$  and the coefficient  $\alpha$  has dimensions  $\text{cm}^3\text{-sec}$ .

In our first assessment of the population density,  $N_y^s$ , we will neglect the induced decay rate term  $\kappa N_y^s$  in (4). Multiplying the resulting equation through by the integrating factor  $e^{\nu_s t}$ , we obtain

$$\frac{d}{dt}(N_y^s e^{\nu_s t}) = \sum_l f_l^s \nu_l N_l(0) e^{-\nu_l t} e^{\nu_s t} \quad (6)$$

where we have set,

$$N_l(t) = N_l(0)e^{-\nu_l t}$$

Integrating (6) gives

$$N_y^s(t) = N_y^s(0)e^{-\nu_s t} + \sum_l \frac{f_l^s \nu_l N_l(0)}{(\nu_s - \nu_l)} [e^{-\nu_l t} - e^{-\nu_s t}] \quad (7)$$

In typical cases,  $\nu_s \gg 1\text{Hz}$  and  $\nu_l \ll 1\text{Hz}$ . Thus (7) reduces to

$$N_y^s(t) = N_y^s(0)e^{-\nu_s t} + \sum_l \left( \frac{\nu_l}{\nu_s} \right) f_l^s N_l(0) [1 - e^{-\nu_s t}] \quad (8)$$

If initially there are no excited states of  $y$ , then (8) further reduces to

$$N_y^s(t) = \sum_l \left( \frac{\tau_s}{\tau_l} \right) f_l^s N_l(0) [1 - e^{-\nu_s t}] \quad (9)$$

where  $\tau \equiv \nu^{-1}$  represents decay time. The time-dependent factor in (9) reaches its asymptotic value in approximately  $\tau_s$  seconds. Population of this excited state would, of course, be further diminished by stimulated decay.

In the following sections the preceding relation will be applied to some specific cases.

### 5. Decay of First Excited State

Let us consider the decay scheme<sup>8</sup> to the stable nucleus  ${}^{161}_{66}\text{Dy}$ . The lasing frequency and wavelength that would ensue, were this scheme to prove effective, are given by

$$h\nu = 25.65 \text{ KeV}$$

$$\nu = 6.2 \times 10^{18} \text{ Hz}$$

$$\lambda = 0.483\text{\AA}$$

which are seen to fall in the X-ray domain.

From Fig. 2 we see that the most promising source for filling the  $s_1$  states is the EC decay of  ${}_{67}\text{Ho}^{161}$ . For this process we find that

$$\frac{\nu_l}{\nu_s} = \frac{\tau_s}{\tau_l} = \frac{29\text{ns}}{2.5\text{h}} = 3.2 \times 10^{-12}$$

which, with the expression (9) is seen to render the proposed scheme unfeasible. A similar conclusion is evident for the predominance of the decay schemes (1) and (2).

## 6. Isomeric Decay

Our second approach emerges for like schemes involving very long-lived isomeric nuclear states. We direct our attention to the decay scheme leading to  ${}_{77}\text{Ir}^{191}$  shown in Fig. 3.

The conjectured lasing frequency and wavelengths have the values ( $s_3 + s_2$ )

$$\nu = 1.1 \times 10^{20} \text{ Hz}$$

$$\lambda = 0.028 \text{ \AA}$$

which again fall in the X-ray domain. Referring to the data<sup>5</sup> given in Fig. 3, we see that

$$\frac{\nu_l}{\nu_3} = \frac{\tau_3}{\tau_l} = \frac{4.95\text{s}}{15\text{d}} = 3.8 \times 10^{-6}$$

The infinitesimal population growth of  $s_3$  excited states may possibly be enhanced through increasing the concentration of the 'pumping' parent nuclei  ${}_{76}\text{Os}^{191}$ . For example, if the device is of the order of  $\text{lm}^3$ , then a concentration of this isotope far in excess of  $10^6/\text{m}^3$  would serve to enhance the

smallness ratio  $\nu_2/\nu_3$ . Thus this conjectured lasing model may find application in astrophysical configurations as a possible model for X-ray emission.<sup>12</sup>

### 7. Multipole Fields and Selection Rules

Such nuclear radiative transitions as described above, are known to conserve parity, and we write<sup>13</sup>

$$\Pi_i = \Pi_f \Pi_r$$

where  $\Pi_{i,f}$  are parities of initial and final states, respectively. The parity of the radiation field is written  $\Pi_r$ , which in turn obeys the rules:

$$\begin{aligned} \text{for E multipole fields, } \Pi_r^E &= (-1)^l \\ \text{for M multipole fields, } \Pi_r^M &= (-1)^{l+1} \end{aligned} \quad (11)$$

For the  $s_3 \rightarrow s_2$  transition described above,  $j_i = 11/2$ ,  $j_f = 5/2$ . The possible  $l$ -values of the emitted multipole radiation field are 10, 11

$$\left| 11/2 - 5/2 \right| \leq l \leq \left| 11/2 + 5/2 \right|, \quad 3 \leq l \leq 8$$

Thus the lowest order multipole field which conserves parity is E3 (electric octipole) as indicated in Fig. 3.

The time-averaged power radiated per unit solid angle, for a pure multipole of order  $(l, m)$  is given by<sup>14</sup>

$$\begin{aligned} \frac{dP(l, m)}{d\Omega} &= \frac{c |a(l, m)|^2}{8\pi k^2} |X_{lm}|^2 \\ |X_{lm}|^2 &= \frac{1}{l(l+1)} \left[ \frac{1}{2}(l-m)(l+m+1) |Y_{l, m+1}|^2 \right. \\ &\quad \left. + \frac{1}{2}(l+m)(l-m+1) |Y_{l, m-1}|^2 + m^2 |Y_{lm}|^2 \right] \end{aligned} \quad (12)$$

where  $a(l, m)$  is a coefficient independent of angle.

The cross section for this process is obtained<sup>2,12</sup> by dividing the radiated power per solid angle (12) by the normalized incident flux  $c/4\pi[\text{erg}/\text{cm}^2 - \text{s}]$ . We find

$$\frac{d\sigma}{d\Omega} = \frac{|a(l, m)|^2}{2k^2} |X_{lm}|^2 \quad (13)$$

For the octapole field ( $l = 3$ ), the azimuthal  $m$ -number may have values,  $m = 0, \pm 1, \pm 3$ . For the azimuthally symmetric state ( $l = 3, m = 0$ ), (12) gives

$$|X_{30}|^2 = \frac{1}{2} \left[ \left( \frac{1}{4} \sqrt{\frac{21}{4\pi}} \right)^2 \sin^2 \theta (5 \cos^2 \theta - 1)^2 \right] = \frac{1}{32} \sqrt{\frac{21}{4\pi}} \psi(\theta) \quad (14)$$

This field falls to zero at  $\sin^2 \theta = 0$  and at  $\cos^2 \theta = 1/5$  ( $\theta = 63.43^\circ$ ). It is maximum at  $\cos \theta = 0$  [ $\psi(\theta) = 1$ ] and at  $\cos^2 \theta = 11/15$  ( $\theta = 31.09^\circ$ ) [ $\psi(\theta) = 1.90$ ]. Thus  $|X_{30}|^2$  has its larger maximum at  $\theta = 31.09^\circ$  and smaller maximum at  $\theta = 90^\circ$ . Apart from a matrix factor, the angular distribution of radiated power (12) is the same as that which emerges in the corresponding quantum mechanical evaluation<sup>16,17</sup>.

## 8. Conclusion

We have described two nuclear decay schemes appropriate to heavier nuclei which may give rise to lasing in the  $\gamma$  and X-ray bands. Parent nuclei undergoing either  $\beta$ -decay or electron capture serve as a pumping mechanism which supplies certain long-lived excited states. When applied to two specific cases, a derived expression for the population of these states gives infinitesimal values. The resulting expression also indicates that this population may be enhanced by increasing the concentration of parent nuclei.

## Acknowledgments

Fruitful discussions with my (RLL) colleague David Clark on these topics are gratefully acknowledged. This research was supported in part under contract AFOSR 78-3574.

## Figure Captions

### Figure 1

Decay schemes to excited states of  $Z+1Y^M$ .

### Figure 2

Decay schemes to  ${}^{161}_{66}\text{Dy}$ .

### Figure 3

Decay schemes to  ${}^{191}_{77}\text{Ir}$ .

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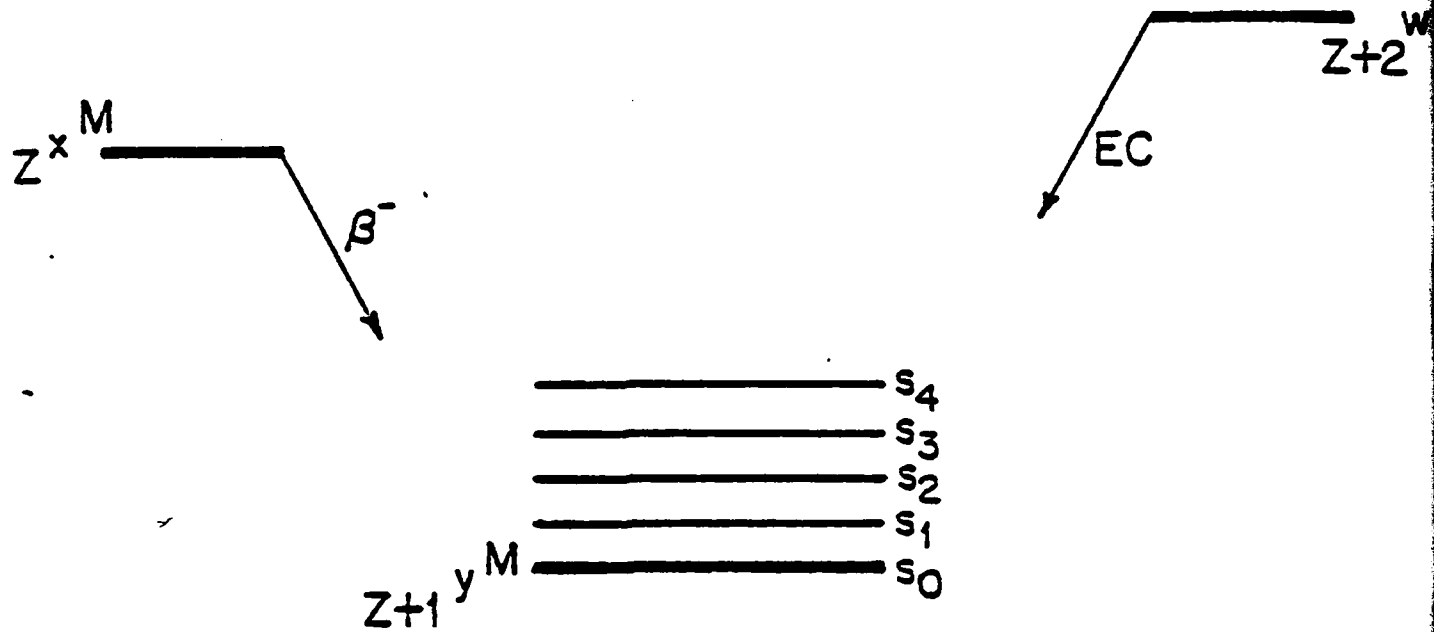


Figure 1

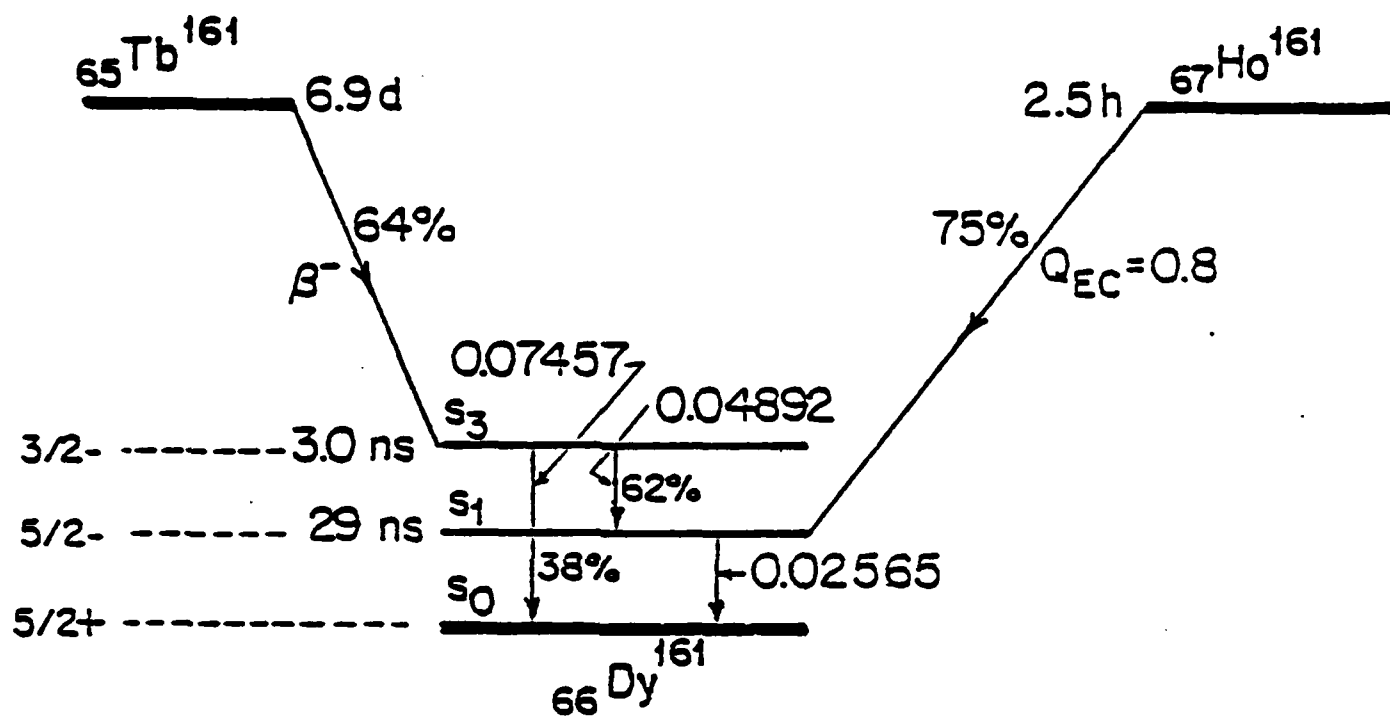


Figure 2

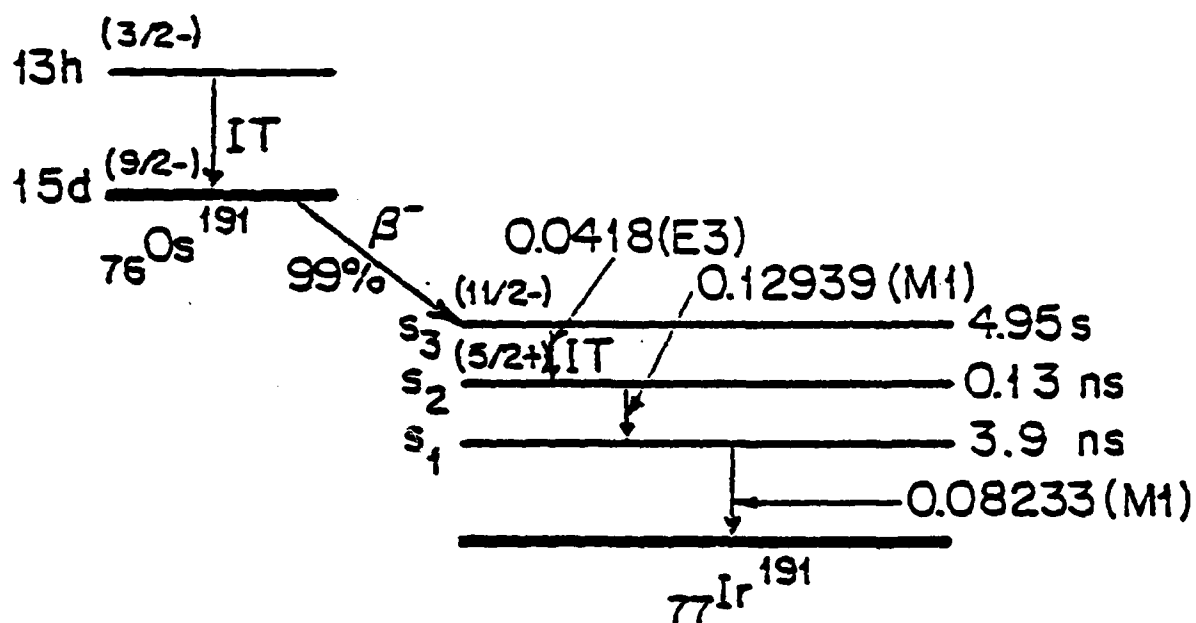


Figure 3

